

歐拉 (Leonhard Euler, 1707 – 1783) 贊辭

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謹於公元 2007 年七月三日，在俄羅斯聖彼得堡之亞歷山大涅夫斯基修道院墓園，歐拉墓前恭讀。

爲了紀念偉大瑞士數學家歐拉的三百週年誕辰，我們必須將時光回溯到十八世紀的歐洲，回到那堪譽爲「歐拉的年代」的歲月。名數學家，同時也是歐拉的仰慕者，特魯思德爾（Clifford Truesdell）曾經說過：「研究歐拉的工作，就是環視十八世紀中葉的整體科學，以及當時大部分的人類智識成就。」¹在那個年代，歐洲最重要的學術活動與科學進展，大多發生在寥寥可數的幾個皇家科學院，而非大學校園。而且，當時人們對科學的追求不受國籍與地理的限制，所以，最優秀的學者從歐洲各地聚集至同一所科學院併肩工作。那些學者的其中之一，就是「無與倫比的歐拉」（借用約翰·柏努利（Johann Bernoulli）的話）。

歐拉二十歲離開家鄉。接 的五十六年，他在聖彼得堡的俄羅斯科學院與柏林的普魯士科學院作出超大量的貢獻。法國數學家與物理學家阿哈果（François Arago）曾說：「歐拉輕鬆地進行計算，就像人呼吸，或是老鷹御風飛行。」另一位法國哲學家與數學家康多塞侯爵（Marquis de Condorcet）也在給歐拉的贊詞中提到：「他停止計算與呼吸了。」那是發生在 1783 年九月十八日的晚上，歐拉在聖彼得堡的宅邸中風逝世。今日，我們來到他的墓前表達敬意。

歐拉以他於科學上極爲豐富的貢獻著稱於世，這些貢獻無論在質在量，歷史上均無人能出其右。有個關於他的趣聞是這麼說的：歐拉撰寫的論文，由於完成的速度比較出版的速度還要快，所以，往往較晚完成的作品更先發表，結果就是，先出版的作品的內容竟然比後出版的更豐富而且更完善！

十八世紀後四分之三的日子裡，在數學、數學物理與工程力學方面的研究，約有三分之一出於歐拉之手筆。歐拉所有著作的現代修訂版（*Opera Omnia*）從 1911

¹ C. Truesdell, “Leonhard Euler, Supreme Geometer”, in C. Truesdell, *An Idiot's Fugitive Essays on Science* (New York : Springer-Verlag, 1984), pp. 337-379.

年開始編纂，至今尚未完成，但到 1994 年已有近八十巨冊面世。1983 年《數學雜誌》（*Mathematics Magazine*）出版歐拉紀念專刊，裡面列出了四十四項以歐拉命名的數學名詞或定理。他是個近乎全才的非凡研究者，不論在連續（continuous）或離散（discrete）的領域中都有貢獻，因此，他是一位真正的「具體」（con-crete）²數學專家！³歐拉的影響不僅限於十八世紀，直到現代，數學家仍受用無窮。舉兩個例子。歐拉有名的哥尼斯堡七橋問題的解答在 1960 年代導致「中國郵差問題」上的「管梅谷-Edmonds」演算法。在 1970 年代，它也導致「（歐氏）旅行銷售員問題」上的「克里斯費德（Christofides）」演算法，找到不差於二分三最優解的方案。第二個例子是關於數論。歐拉計算了一個有名的無窮級數，將連續整數平方的倒數相加。它被稱為 $\zeta(2)$ ，答案是 $\pi^2/6$ 。歐拉還成功地計算所有 n 為偶數的 $\zeta(n)$ 。直到二百四十年後，艾裴里（Roger Apéry）才証明了 $\zeta(3)$ 是無理數，但時至今日仍無人算出 $\zeta(3)$ 的值。最近，數學界重新對計算奇數 n 值的 $\zeta(n)$ 大感興趣，並且開始有了進展。

身為一位作者與教師，歐拉以論述清晰而聞名，但除此之外，他更渴望與讀者分享他如何做出那些發現。⁴難怪拉普拉斯（Pierre-Simon Laplace）說：「要閱讀歐拉，要閱讀歐拉。他是我們所有人的老師！」歐拉最關心的，在於發現時的奇妙與發現過程的闡述，而不在意是他自己抑或由別人做出這項發現。已故俄羅斯數學史家尤許克維奇（Adolf P. Youshkevitch）認為歐拉是一位好心的人，毫不嫉妒他人，而且還借了馮特內爾（Bernard le Bouyer de Fontenelle）寫給萊布尼茲（Gottfried Wilhelm Leibniz）的有名贊詞中的話，來表揚歐拉這種高尚的品格：「他樂於見到，他在別人花園中灑下的種子，能夠開花結果。」⁵

另一個叫我們景仰歐拉的原因，是他在逆境中仍然有豐盛的生命。歐拉二十八歲身染重病，導致三十一歲時失去右眼視力。在五十九歲時再度染病，失去部份左眼視力，且在六十四歲時左眼視力更加退化，在他辭世前十二年一直完全失明。同時在六十四歲，一把大火燒毀了他的房屋與財產，兩年後他的首任妻子也過世。歐拉有十三個兒女，但八位夭折。一次又一次的厄運並未擊倒歐拉，他反而更加專注

² 譯按：此處為雙關語，「concrete」一字是由「continuous」的前半與「discrete」的後半組成，意指歐拉在連續數學與離散數學均有貢獻，且有「具體」的應用。

³ 《具體數學》（*Concrete Mathematics*）是 R.L. Graham、D.E. Knuth、O. Patashnik 所寫的一本書，1989 年 Addison-Wesley 出版。三位作者說他們猶未有足夠勇氣採用 “distinuous mathematics” 這個名詞！

⁴ 除了計算 $\zeta(2)$ 之外，有很多的例子可以在下面的論文中找到：M.K. Siu, “Euler and heuristic reasoning: mathematical thinking and history of mathematics”, in *Learn From the Masters! Proceedings of the Kristiansand Conference on History of Mathematics and its Place in Teaching*, August 1988, edited by F. Swetz et al, Mathematical Association of America, Washington D.C., 1995, 256-275; 279-282.

⁵ A.P. Youshkevitch, “Euler”, in *Biographical Dictionary of Mathematicians*, Volume 2 (from *Dictionary of Scientific Biography*, Volume 4), edited by C.C. Gillispie, Scribner's, New York, 1970-80, 736-753.

於工作。歐拉的研究成果，幾乎有半數是在他將近六十歲之後完成的。

《詩經》中有句話，「高山仰止，景行行止」，正足以說明我們對歐拉的景仰。歐拉是個天才，少有人能望其項背。但我們能學習他對生命、工作與研究的熱情，他那永不滿足的好奇心及探索精神，他要達到深刻理解的決心，他的勤奮，他的謙遜，他的慷慨，以及他用平靜面對困頓的堅強心志。這些都是我們要向他學習之處。

在歐拉的家鄉（瑞士巴塞爾附近的里恩（Riehen）），一塊紀念歐拉的匾額上，簡要地總結了這位簡單但偉大的學者的一生：“Er war ein grosser Gelehrter und ein gütiger Mensch”（他是一位偉大的學者，也是一位善良的人）。

蕭文強

2007 年七月三日

於聖彼得堡

Eulogy of Leonhard Euler (1707 – 1783)

Respectfully read before the tomb of Euler at the Alexander Nevsky Monastery Cemetery in St. Petersburg on July 3, 2007.

To commemorate the three hundredth anniversary of the great Swiss mathematician Leonhard Euler we would have to transport ourselves to the Europe in the eighteenth century, which can be justly hailed as ‘the Age of Euler’. As the late Clifford Truesdell, himself a renowned mathematician and a life-long admirer of Euler, put it, “To study the work of Euler is to survey all the scientific life, and much of the intellectual life generally, of the central half of the eighteenth century.”⁶ It was a time when academic activities and scientific progress in Europe took place most prominently in the few royal academies rather than in the many universities. It was also a time when no national distinction or geographical boundary was recognized in the pursuit of science, so that eminent scholars came from different parts of Europe to study and work side by side at the same academy. Among them was the “incomparable Leonhard Euler” (in the words of Johann Bernoulli).

⁶ C. Truesdell, Leonhard Euler, Supreme Geometer, in C. Truesdell, *An Idiot's Fugitive Essays on Science*, Springer-Verlag, New York, 1984, 337-379.

Euler left his homeland at the age of twenty and spent the next fifty-six incredibly productive years in the Russian Academy in St. Petersburg and the Prussian Academy in Berlin. The French mathematician-physicist François Arago said of him, “Euler calculated without apparent effort, as men breathe, or as eagles sustain themselves in the wind.” The French philosopher-mathematician Marquis de Condorcet said in a eulogy of Euler, “He ceased to calculate and to breathe.” That happened on the evening of September 18, 1783 when Euler suffered from a stroke at his home in St. Petersburg. Today we come in homage to his burial ground.

Euler is well known for his prolific output of an order of magnitude unparalleled in history, both in quality and quantity. There is this amusing story that the accumulation of the stack of his finished memoirs increased more rapidly than the rate these memoirs went to press, so that very often they were published in the reversed order of completion with the strange consequence that the content of a published memoir was more extended and improved than that of another that appeared later!

Approximately one third of the research in mathematics, mathematical physics and engineering mechanics published in the last three-quarters of the eighteenth century was authored by Euler. The modern revision of his collected works (*Opera Omnia*) began in 1911 and is not yet finished, with already near to eighty volumes published by 1994. In 1983 a special issue of *Mathematics Magazine* dedicated to his memory listed forty-four items of mathematical terms or theorems that bear his name. His versatility as an all-round researcher is phenomenal, both in the continuous and in the discrete, so that he was truly an expert in “con-crete” mathematics!⁷ The influence of Euler goes far beyond the eighteenth century and into the era we live in. Just cite two examples. His idea on the famous seven bridges of Königsberg led to the Guan-Edmonds algorithm on the Chinese Postman Problem in the early 1960s. It also led to the efficient Christofides algorithm in the mid 1970s that yields a solution to the (Euclidean) Travelling Salesman Problem no worse than three-halves of the optimal solution. The second example is in number theory. Euler calculated the famous infinite sum of the reciprocal of consecutive squares, also known as $\zeta(2)$, to be π squared over six, and treated successfully $\zeta(n)$ for n even. Only more than two hundred and forty years later did Roger Apéry prove that $\zeta(3)$ is irrational, but nobody yet knows what $\zeta(3)$ is. In

⁷ *Concrete Mathematics* is the title of the book written by R.L. Graham, D.E. Knuth, O. Patashnik, published by Addison-Wesley in 1989. The authors say that they are not bold enough to try “distinuous mathematics”!

recent years both interest and progress are rekindled in the investigation of $\zeta(n)$ for odd n .

As an author and teacher Euler is noted for his clarity in exposition, but more than that he was eager to share with his readers his ideas that led to those discoveries.⁸ No wonder Pierre-Simon Laplace said of him, “Read Euler, read Euler. He is the master of us all.” His primary interest was in the wonder of discovery and its explication, but cared little whether he or somebody else made the discovery. The late Russian historian of mathematics, Adolf P. Youschkevitch, described Euler as a well disposed man not given to envy, and further borrowed from the famous eulogy by Bernard le Bouyer de Fontenelle on Gottfried Wilhelm Leibniz to highlight this noble trait of Euler, “He was glad to observe the flowering in other gardens of plants whose seeds he provided.”⁹

One more thing about the man that earns our admiration is the way he led a fruitful life despite adversity. He was seriously ill at twenty-eight that led to loss of sight in his right eye at thirty-one, then was struck by illness again at fifty-nine that led to partial loss of sight in his left eye, and at sixty-four became totally blind for the last twelve years of his life. At sixty-four he was further hit by misfortune in losing his house and properties in a fire, followed two years later by the death of his first wife. Euler had thirteen children, but eight died in infancy. He took all the misfortune in stride and was so dedicated to his work that almost half of his research output was produced after he was near to sixty.

In the ancient Chinese Book of Odes one reads the verses that have come to connote the noble character we admire and the virtuous deed we emulate : “高山仰止, 景行行止 (The high mountain I look up at it. The great road I travel on it).” Euler was a genius, whose height very few can hope to reach. But we can all learn from his great zest for life, work and study, his insatiable curiosity to know and to probe, his determination to procure deeper and deeper understanding, his industry, his modesty, his generosity, and his toughness in facing adversity with tranquility. These are qualities we try to strive for.

⁸ Many more examples, besides the computation of $\zeta(2)$, can be found in M.K. Siu, Euler and heuristic reasoning; Mathematical thinking and history of mathematics, in *Learn From the Masters! Proceedings of the Kristiansand Conference on History of Mathematics and its Place in Teaching*, August 1988, edited by F. Swetz et al, Mathematical Association of America, Washington D.C., 1995, 256-275; 279-282.

⁹ A.P. Youschkevitch, Euler, in *Biographical Dictionary of Mathematicians*, Volume 2 (from *Dictionary of Scientific Biography*, Volume 4), edited by C.C. Gillispie, Scribner's, New York, 1970-80, 736-753.

The inscription on a plaque put up in his memory in his hometown (Riehen near Basel) sums up succinctly the life of this simple yet great man, “Er war ein grosser Gelehrter und ein gütiger Mensch (He was a great scholar and a kind man).”

SIU Man Keung

July 3, 2007.

St. Petersburg
